Ray-Tracing in a 3-D Wind Field for Prediction Purposes of Shooting Noise, Part II

Jürgen Zangers[1], Karl-Wilhelm Hirsch[1]

[1] *Institut für Lärmschutz, D-40489 Düsseldorf, Germany, Email: mail@ifl-acoustics.de*

Introduction

Part I discusses the need of a sound propagation model for shooting sounds that can consider a general atmosphere in 3D. The most important parameter of the atmosphere (in this connexion) is the 3D wind vector field to simulate any state of the atmosphere of a realistic propagation situation.

The basic concept of the model relies on the ray-tracing scheme introduced by Pierce [1] and a finite element model that propagates the acoustical energy of the shooting blasts through so-called aku-rays.

The numerical implementation of this concept requires sophisticated procedures. (i) The integration method and the step size increments used for integration are critical for accuracy and computation time. (ii) Due to the ill-conditioned non-linear differential equations, it is necessary to generate a mesh on the sphere that is fine enough to solve the equations on the propagating wave front accurately. (iii) In order to maintain spatial resolution for long-range propagation, a splitting process is required to divide rays into a collection of sub-rays. (iv) The reflection at the ground needs complex numerical concepts to describe the geometry of the elements close to the ground.

Ray-tracing equations

The basic equations for the propagation of sound rays are the ray-tracing equations derived by Pierce [1].

$$(1)\quad \left.\frac{dx_{p_i}}{dt}\right|_{t=t_0} = \left.\left[\frac{c^2 s_i}{\Omega} + v_i\right]\right|_{\vec{x}_p(t=t_0)}$$

$$(2)\quad \left.\frac{ds_i}{dt}\right|_{t=t_0} = \left.\left[-\frac{\Omega}{c}\frac{\partial c}{\partial x_i} - \sum_{j=1}^{3} s_j \frac{\partial}{\partial x_i} v_j\right]\right|_{\vec{x}_p(t=t_0)}$$

for i = 1, 2, 3

with $\vec{s} = \dfrac{\vec{n}}{c + \vec{v}\cdot\vec{n}}$ and $\Omega = 1 - \vec{v}\cdot\vec{s}$

whereas:

$\vec{x}$ position vector of a point in space

$\vec{x}_p$ position vector of a point on wave front

$\vec{n}$ unit vector normal to the wave front (depends on $\vec{x}_p$ und t)

c sound speed (depends on $\vec{x}$ and possibly t)

$\vec{v}$ wind vector (depends on $\vec{x}$ and possibly t)

$\vec{s}$ wave-slowness vector (depends on $\vec{x}_p$ and t)

x_{p_k}, s_k, v_k, x_k Cartesian coordinates of the appropriate vectors (k = 1, 2, 3)

These equations are a coupled system of non-linear partial differential equations of first order. They are amenable to standard numerical techniques of integration (but turned out to be ill conditioned). If a ray position $\vec{x}_p$ and a wave-slowness vector $\vec{s}$ are specified at time t_0, the equations can be integrated in time to determine $\vec{x}_p$ and $\vec{s}$ at any subsequent instant. The choice of an appropriate method is determined by its global error order and the necessary computation time. Rather simple methods like the "Euler-Cauchy method" [2] are very fast but result in unpredictable uncertainties for long-range propagation. Dushaw and Colosi [3] investigate the ray-tracing in water and found that the "Classical Runge-Kutta Method" is the most efficient way to perform those calculations. This conclusion is shared by Press [4] and agrees with our experience. In addition to the integration method, the step size increments Δt are critical for accuracy and computation time. For the procedure here it is appropriate to choose the step size increments as a function of the partial derivates at the right hand sides of equation (1) and (2). In our calculation it varies between 0.001 s and 0.01 s.

The concept of pentahedrons (tents)

The origin of the ray-tracing is a directional spherical source. The surface of this sphere builds up the wave front of the sound wave at the initial time t = 0. The calculation starts on a mesh on this surface. Every grid point $\vec{x}_p(t=0)$ of the triangulation is then propagated according equation (1) and (2) with the appropriate step size increment. The parametric curve $\vec{x}_p(t)$ is called "a ray". Three adjacent rays constitute the edges of the pentahedron; the points of equal propagation time establish the vertices of the two triangles. These triangles together with their connecting lines build up the geometric properties of a pentahedron (tent). Additional features are the neighbourhood relationships, enclosed energy and a lot more parameters for numerical treatment. In the following, these pairs of triangles (and so the tents) are propagated.

The fig. 1 and 2 illustrate this propagation and the basic idea.

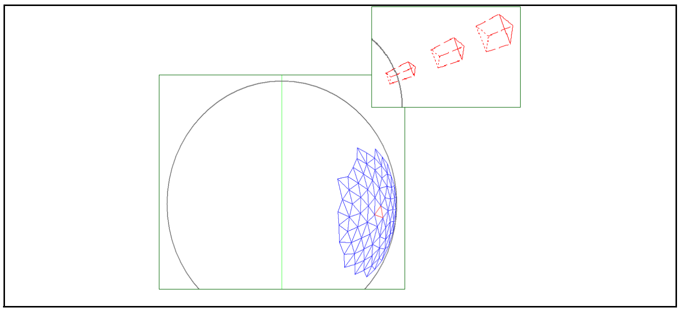

Figure 1: Starting with a mesh on the sphere the tents are propagated according equation (1) and (2)

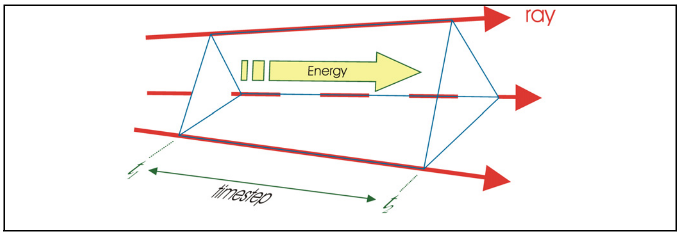

Figure 2: The transfer of acoustical energy is realized through the propagation of tents (blue pentahedron), which consist of 2 triangles together with their connecting lines. The vertices of the triangles are the propagated grid point of the mesh on the sphere (see fig. 1).

Splitting process

In order to maintain spatial resolution for long-range propagation, a splitting process is required to divide aku-rays into a collection of sub-aku-rays. This process bases on 2D and 3D interpolating surface splines [2]. These methods require more points of the wave front than the three vertices of the triangles. Therefore 10 new points are attached to every mesh triangle on the sphere and propagated like the grid points of the triangulation ("spline-rays") see fig. 3.

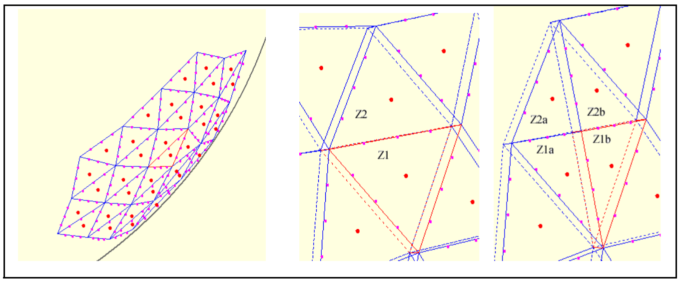

Figure 3: Mesh on the sphere with attached points (red ones) and two adjacent tents together with their attached spline-rays before and after splitting

Reflection at the ground

Reflection at the ground is not as simple as it looks like at first sight. Firstly, the three rays of a tent are reflected at the ground independently of each other. Therefore, it could happened that at a certain time t one of its rays is already reflected (possibly a lot of times) while another ray was not yet reflected. In detail it has to be considered if rays were reflected before the actual propagation step and, if so, which rays are reflected now and in which order. A discussion of numerical details is beyond the scope of this article. Fig. 4 shows the simplest case.

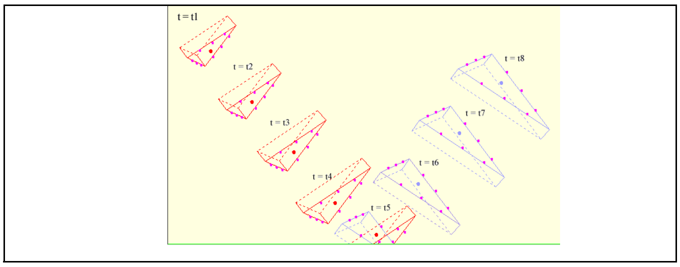

Figure 4: Simple reflection at the ground.

Sound exposure level

If an aku-ray penetrates a receiver area, the fraction E of its energy - determined by the shared area of the aku-ray and the receiver area – which is transmitted to the receiver is calculated. With E and the averaged value A of the triangle areas an averaged energy flux density $\Xi_E = E/A$ is calculated. Thus during the time Δt the receiver is penetrated by the sound intensity $I = \Xi_E / \Delta t$. The sound exposure level yields

$$L_p = 10 \cdot \lg \left(\frac{\int_0^{t_E} I \cdot dt}{10^{-12}\, Ws/m^2} \right)$$

Whereas t_E is the time while the receiver area is penetrated by aku-rays.

Remarks

As mentioned in Part I, ray-tracing equations are normally used to set up (2D) particle models, describing the spreading of sound through a large number of sound particles that are released at the source and collected at the receiver sites. We could show that this method is not favourable to large distances in 3D. Firstly, it turns out that even in absence of wind the $1/r^2$ - law is not satisfied (in contrast to 2D calculations). Secondly, numerical tests (with a density of 43,000,000 particles per m^2 on the unit sphere) for long-range propagation (3 km and more) indicated that the number of collected particles at the receiver site is too small to have any statistical significance.

At the present model state the results of the program are noise maps. These are calculated for a quadratic area of 100 square kilometres with a spatial resolution of 100 m x 100 m. The fineness of the mesh on the sphere varied between 50,000 triangles and 250,000. The computation time (on a computer with 2.4 GHz CPU and 1 GB RAM) ranges from 6 h to 35 h depending on some other parameters as well.

Due to the ill conditioned ray-tracing equations slight variations of the initial mesh cause dramatic changes in the ray paths and thus in the shape of the tents. The corresponding variations of the predicted sound exposure levels are a measure for the "numerical accuracy". The "numerical accuracy" varied between 0.2 dB and 2 dB in down wind direction and 2 to 7 dB in up wind direction subject to the fineness of the initial mesh and the splitting parameters. The small values are reached with a fine mesh (250,000 triangles).

References

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[4] Press, W., H., et al., Numerical Recipes in Pascal, Cambridge University Press, 1989, Chapter 15

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